Foundation for success

Unified International
Mathematics Olympiad

## UNIFIED INTERNATIONAL MATHEMATICS OLYMPIAD (UPDATED)



KEY

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D | C | D | B | A | D | C | B | B | A |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| A | D | D | A | B | C | A | A | C | B |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| C | D | C | B | A | A | B | A | D | C |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| $(\mathrm{~A}, \mathrm{~B}, \mathrm{D})$ | $(\mathrm{A}, \mathrm{B}, \mathrm{D})$ | $(\mathrm{A}, \mathrm{B}, \mathrm{C})$ | $(\mathrm{A}, \mathrm{B}, \mathrm{C})$ | $(\mathrm{B}, \mathrm{C}, \mathrm{D})$ | B | C | C | C | D |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| C | A | C | D | D | D | B | B | A | C |

## EXPLANATIONS

## MATHEMATICS - 1

1. (D) $x=60^{\circ} \quad[\because$ corresponding angles $]$
$x+y=180^{\circ}$
$y=180^{\circ}-60^{\circ}=120^{\circ}$
2. (C) Let the other number be $x$

Given $-36+x=23$
$x=23+36=59$
3. (D) $10^{2}=8^{2}+6^{2} \Rightarrow 6 \mathrm{~cm}, 8 \mathrm{~cm}$ and 10 cm are the sides of a right angled triangle
$\therefore \quad$ Area of right angled triangle $=\frac{1}{2} \times$ product of perpendiculars
$=\frac{1}{2} \times 6 \mathrm{~cm} \times 8 \mathrm{~cm}=24 \mathrm{~cm}^{2}$
4. (B) Given in $\triangle A B C, A B=A C \Rightarrow \angle C=\angle B=50^{\circ}$ $\therefore \angle A=180^{\circ}-\angle B-\angle C=80^{\circ}$
5. (A) SSS criterion is used to construct equilateral triangle
6. (D) If $\frac{x}{y}<1 \Rightarrow \frac{y}{x}>1$
7. (C) Sum of first ten composite numbers = 112

Mean of sum of first ten composite numbers $=112 / 10=11.2$
8. (B) To construct a triangular pyramid it is required 4 equilateral triangles
9. (B) $x+y+y+z+z+x=8+5+7=20$
$2 x+2 y+2 z=20$
$2(x+y+z)=20$
$x+y+z=\frac{20}{2}=10$
$8+z=10$
$z=10-8=2$
10. (A) Let CP of each article be $₹ x$
$\therefore \quad$ CP of 12.5 article $=₹ 12.5 x$
Given SP of 10 article $=₹ 12.5 x$
$\therefore \quad$ SP of 12.5 article $=\frac{₹ 12.5 x}{10}$
$=$ ₹ $1.25 x$
$\therefore \quad$ Gain $=\mathrm{SP}-\mathrm{CP}=1.25 x-x=0.25 x$
$\therefore \quad$ Gain percentage
$\frac{\text { Gain }}{C P} \times=\frac{₹ 0.25 x}{x} \times 100=25 \%$
11. (A) Option ' $A$ ' is a pentagon but remaining three options are quadrilaterals
12. (D) Given $(4 x-7) \mathrm{cm}=(2 x+5) \mathrm{cm}$
$4 \mathrm{x}-2 \mathrm{x}=5 \mathrm{~cm}+7 \mathrm{~cm}$
$2 \mathrm{x}=12 \mathrm{~cm}$
$x=6 \mathrm{~cm} \Rightarrow 2 \mathrm{x}+5=17 \mathrm{~cm}$
13. (D) Middle odd number $=\frac{55}{5}=11$

Next odd number $=11+2=13$
Largest odd number $=13+2=15$
14. (A) Let principal be ₹ x

Amount $=\frac{41}{40} \mathrm{x}$
$\mathrm{I}=\mathrm{A}-\mathrm{P}=\frac{41 x}{40}-x$
$\mathrm{I}=\frac{x}{40}$

But $\mathrm{I}=\frac{\mathrm{PTR}}{100}=\frac{x}{40}$
$\frac{x \times \mathrm{R} \times \frac{1}{4}}{100}=\frac{x}{40}$
$R=10 \%$
15. (B) $\ln \triangle \mathrm{ADC}, 70^{\circ}+\angle \mathrm{C}+\angle \mathrm{ADC}=180^{\circ}$
$\angle C+\angle A D C=180^{\circ}-70^{\circ}=110^{\circ}$
But $y=\angle \mathrm{C}+\angle \mathrm{ADC}=110^{\circ}$
16. (C) Given $(\text { side })^{2}=196 \mathrm{~cm}^{2}=(14 \mathrm{~cm})^{2}$
$\therefore \quad$ Side $=14 \mathrm{~cm}$
$\therefore \quad$ Diameter of the circle $=14 \mathrm{~cm}$
$\therefore \quad$ Radius $=\frac{d}{2}=7 \mathrm{~cm}$
Area of the circle $=\pi r^{2}=\frac{22}{7} \times 7 \times 7 \mathrm{~cm}^{2}$
$=154 \mathrm{~cm}^{2}$
Remaining area $=196 \mathrm{~cm}^{2}-154 \mathrm{~cm}^{2}=$ $42 \mathrm{~cm}^{2}$
17. (A) $\frac{6}{7}=0.857, \frac{7}{9}=0.777$
$\therefore \frac{95}{112}=0.848, \frac{99}{112}=0.883$,
$\frac{3}{4}=0.75, \frac{97}{112}=0.866$
$\therefore \quad 0.848$ lies between $0.857 \& 0.777$
$\therefore \frac{7}{9}<\frac{95}{112}<\frac{6}{7}$
18. (A) Given figure has 4 lines of symmetry

19. (C) The above given table is in the form of $y$ $=9 x-2$
20. (B) $\mathrm{LHS}=\frac{1}{3} y^{2}-\frac{4}{7} y+11-\frac{1}{7} y$

$$
\begin{aligned}
& +3-2 y^{2}-\frac{2}{7} y+\frac{2}{3} y^{2}-2 \\
= & \left(\frac{1}{3} y^{2}-2 y^{2}+\frac{2}{3} y^{2}\right)+\left(\frac{-4}{7} y-\frac{1}{7} y-\frac{2}{7} y\right) \\
& +(11+3-2) \\
= & \left(\frac{-y^{2}-6 y^{2}+2 y^{2}}{3}\right)+\left(\frac{-4 y-y-2 y}{7}\right)+12 \\
& =\frac{-3 y^{2}}{3}-y+12 \\
= & \left(-y^{2}-y+12\right)
\end{aligned}
$$

21. (C) Given $A B \| C D$
$\Rightarrow \angle \mathrm{DAB}+\angle \mathrm{ADC}=180 \quad \rightarrow \quad$ (1)
Given $A D \| B C \Rightarrow \angle D$
$\mathrm{AB}+\angle \mathrm{ABC}=180 \quad \rightarrow \quad(2)$
From eq (1) \& (2)
$\angle A D B+\angle A D C=\angle D A B+\angle A B C$
$\therefore \quad \angle \mathrm{ADC}=\angle \mathrm{ABC}$
22. (D) Given area of first square $=36 \mathrm{~cm}^{2}=(6 \mathrm{~cm})^{2}$

$\therefore \quad \mathrm{S}_{1}{ }^{2}=(6 \mathrm{~cm})^{2}$
$\therefore \quad S_{1}=6 \mathrm{~cm}$
Similarty $\mathrm{S}_{2}=8 \mathrm{~cm} \& \mathrm{~S}_{3}=10 \mathrm{~cm}$
$\therefore \quad \mathrm{S}_{1}+\mathrm{S}_{2}+\mathrm{S}_{3}=6 \mathrm{~cm}+8 \mathrm{~cm}+10 \mathrm{~cm}=24 \mathrm{~cm}$
In $\triangle A C D, A C=24 \mathrm{~cm} \& B C=10 \mathrm{~cm} \& \angle C=90$
$\therefore \quad A B^{2}=(24 \mathrm{~cm})^{2}+(10 \mathrm{~cm})^{2}=576 \mathrm{~cm}^{2}+100$ $\mathrm{cm}^{2}=676 \mathrm{~cm}^{2}$
$A B^{2}=(26 \mathrm{~cm})^{2}$
$\therefore \quad A B=26 \mathrm{~cm}$
23. (C) $\frac{a^{2} b c^{3} \times a^{5} b^{4} \times c^{2}}{a^{-7} b^{-8} c^{9} \times a^{-11} b^{12} c^{-13} \times a^{14} b^{-15} c^{-16}}$

$$
\begin{aligned}
& =a^{2+5+7+11-14} \quad b^{1+4+8-12+15} \\
& c^{3+2-9+13+16} \\
& =a^{11} b^{16} c^{25}
\end{aligned}
$$

24. (B)

$$
\begin{aligned}
& \frac{\left(\frac{21 x^{3} y^{3}}{25}\right)}{\frac{7 x^{2} y}{5}}+\frac{\frac{14 x^{3} y}{25}}{\left(\frac{7 x^{2} y}{5}\right)}=\frac{21^{3} x^{3} y^{3}}{25_{5}} \\
& \times \frac{\not 5}{7 x^{2} y}+\frac{14^{2} x^{3} y}{25_{5}^{5}} \times \frac{\not 5}{{ }_{1} 7 x^{2} y} \\
& =\frac{3}{5} x y^{2}+\frac{2}{5} x
\end{aligned}
$$

25. (A) Given $4^{x}-\frac{4^{x}}{4}=24 \quad 2^{2 x}=2^{5}$

$$
\Rightarrow \frac{4^{x} \times 4-4^{x}}{4}=24 \quad 2 x=5
$$

$$
\Rightarrow 4 x \frac{(4-1)}{4}=24 \quad x=\frac{5}{2}
$$

$$
\Rightarrow 4^{x} \times \frac{3}{4}=24
$$

$$
(2 x)^{x}=\left(\not 2 \times \frac{5}{\not 2}\right)^{\frac{5}{2}}
$$

$$
\Rightarrow\left(2^{2}\right)^{x}=24 \times \frac{4}{3}=32=5^{5 / 2}
$$

26. (A) LHS $=\frac{15}{2}+\frac{1}{2} \div \frac{1}{2}$ of $\frac{1}{4}-\frac{2}{5} \times \frac{7}{3}$

$$
\begin{aligned}
& \div \frac{15}{8} \text { of }\left(\frac{7}{5}-\frac{4}{3}\right) \\
& =\frac{15}{2}+\frac{1}{2} \times 8-\frac{2}{5} \times \frac{7}{3} \div \frac{15}{8} \text { of } \frac{1}{15} \\
& =\frac{15}{2}+4-\frac{2}{5} \times \frac{7}{3} \times 8 \\
& =\frac{15}{2}+4-\frac{112}{15}=\frac{225+120-224}{30}
\end{aligned}
$$

$=\frac{121}{30}$
$=4 \frac{1}{30}$
27. (B) Only one line can be drawn through the given point which is parallel to the given line

28. (A) Given
$A: B=0.010 .11=0.01 \times 1000.11 \times 100$ = $1: 11$
$B: C=2.2: 1=2.2 \times 10: 1 \times 100=$ $22^{11}: 10^{5}=11: 5$
$\therefore \quad A: B: C=1: 11: 5$
29. (D) Side of a square

$$
=\frac{\text { Perimeter }}{4}=\frac{140 \mathrm{~m}}{4}=35 \mathrm{~m}
$$

Area of square $=S^{2}=(35 m)^{2}=1225$ sq metres
30. (C) Factors of 48 area $1,2,3,4,6,8,12,16$, 24, 48
$\therefore \quad$ Total factors of $48=10$

## MATHEMATICS - 2

31. $(A, B, D)$

Given $\triangle \mathrm{DCP} \cong \triangle \mathrm{MLA}$
$\angle \mathrm{D}=\angle \mathrm{M}, \angle \mathrm{C}=\angle \mathrm{L} \& \angle \mathrm{P}=\angle \mathrm{A}$
and $C P=\angle A, D P=M A, D C=M L$
32. $(A, B, D)$

Given problem is true for all values of $x$ Except zero
33. $(A, B, C)$

Except option 'D' remaining options are true
34. $(A, B, C)$

Except option ' $D$ ' remaining options are true
35. ( $B, C, D$ ) In $\triangle A B D$,

$$
\begin{aligned}
& 90^{\circ}+30^{\circ}+x=180^{\circ} \\
& x=180^{\circ}-120^{\circ}=60^{\circ} \\
& \ln \triangle \mathrm{BCD}, \text { given } \mathrm{BC}=\mathrm{CD} \\
& \Rightarrow \quad \angle \mathrm{CDB}=\angle \mathrm{CBD}=y \\
& y+y+30^{\circ}=180^{\circ} \\
& 2 y=180^{\circ}-30^{\circ}=150^{\circ} \\
& \\
& y=\frac{150^{\circ}}{2}=75^{\circ} \\
& x+y=60^{\circ}+75^{\circ}=135^{\circ}
\end{aligned}
$$

## REASONING

36. (B) The innermost line is Pink instead of Green.

37. (C) The series follows 2 rules:
(1) The number written in words is reduced by 1 every time. This number is written in words and in numerical form alternately.
(2) The second number increases by 1 every time.

By following these rules, the next term will be THREE-5.
38. (C) After studying the given information carefully. We observe that $D<A<B>C$

B $>\mathrm{C}>\mathrm{A}>\mathrm{D}$
$\therefore B>A$
39. (C) Everytime the shape turn $90^{\circ}$ right. The colours of a shape from top to bottom moves next below shape. Bottom shape colour moves to top position shape.
40. (D)

41. (C)

42. (A)

43. (C) $23-12=11 \times 7=77$
$45-28=17 \times 2=34$
$31-19=12 \times 3=36$
44. (D) Pile, Pine, Pice, Pie, Pen and Pin can be formed.
45. (D)



WINTER


## CRITICAL THINKING

46. (D) 46
$8^{\text {th }}$ Jan $2017 \rightarrow$ Sunday
8th Mar 2017 is $31-8$
$\Rightarrow \quad 23$ days $+28+8$
$\Rightarrow \quad \frac{59 \text { days }}{7} \Rightarrow 3$ odd days
$\Rightarrow$ Wednesday
Statement-1 : Mar 2017 is Wednesday (true)

March April May June July 23 days 30 days 31 days 30 days 8 days

$$
\begin{aligned}
\frac{122 \text { days }}{1} & \Rightarrow 17 \text { weeks } 3 \text { odd days } \\
& \Rightarrow \text { Saturday }
\end{aligned}
$$

Statement-2 : $8^{\text {th }}$ July 2017 is a saturday
Hence both statements are true.
47. (B) 3

According to the given picture, $\mathrm{B}, \mathrm{E}$ and A have four friends means

$1,3,4$ are represents the letters $B, E, A$ $F$ is friend with only $B$ and $E$ F represent number 5 $B$ and $E$ reperesents 4 and 1 Therefore A represent number 3
48. (B) False


Minivan is left most corner
Minivan is not parked between the pickup and sedan

So statement 3 is false.
49. (A) Pig +2 Monkeys $=$ Elephant
(from balance 1)
Kangaroo + Pig = Elephant
(from balance 2)
2 Pigs + Monkey $=$ Elephant + Kangaroo
(from balance 3)

Elephant + Kangaroo $=2$ Pigs + Monkey (3)
Elephant - Kangaroo $=$ Pig (2)

2 (Pig + 2 Monkeys) $=3$ Pigs + Monkey
2 Pigs +4 Monkeys $=3$ Pigs + Monkey
3 Monkey $=$ Pig
Elephant $=3$ Monkeys +2 Monkey
Elephant $=5$ Monkeys
50. (C) There are 2 shortest paths from $A$ to $B$ and there are 3 shortest paths from $B$ to C. Therefore, the number of ways of taking the shortest path from $A$ to $C$ by passing through $B$ is $2 \times 3=6$ ways.

